

Design and optimization of Glass/Epoxy driveshaft for passenger vehicle

Nitish P. Chavan, Rajaram M. Shinde, Suresh M. Sawant

Abstract—Propeller shaft is a mechanical component for transmitting torque and power. The useful work is transmitted from gearbox through propeller shaft, differential, live axle and then finally to wheels. Drive shafts are subjected to torsion and shear stress, equivalent to the difference between the input torque and the load. They must therefore be strong enough to bear the stress, at the same time avoiding too much additional weight as it increases inertia. To allow variations in the alignment and distance between the driving and driven components, conventional drive shafts frequently incorporate one or more universal joints and jaw couplings, which increases weight of drive-shaft. In order to obtain the reduction of the weight of the vehicle without decrease in strength and reliability, composite material technology is an efficient alternative. Here composite driveshaft is manufactured for Maruti Omni car and is compared with steel shaft of the same vehicle. Composite drive shaft design has many variables that are needed to be tuned and optimized. Optimization of composite shaft is done with Teaching Learning Based Optimization Algorithm (TLBO), which is very efficient and requires less computational effort due to absence of algorithm specific parameter.

Keywords—Composite Drive shaft, Teaching Learning based optimization (TLBO), Natural frequency, Critical speed

1 INTRODUCTION

Composite materials are largely used in aerospace, defense, marine, automobile, and many other industries. Composites are light-weight and stiff than other structural materials. A laminated composite material consists of several layers of a composite mixture consisting of matrix and fibers. Each layer may have similar or dissimilar material properties with different fiber orientations under varying stacking sequence. There are many alternatives available while designing a particular composite material application. It is important to consider characteristics of such composite structures, such as the dynamic and buckling characteristics subjected to dynamic loads. For example, when the frequency of the loads matches with one of the resonance frequencies of the structure, large deflections and internal stresses lead to failure of structure components.

A laminate is constructed by stacking a number of laminas in the thickness (z) direction. Each layer is thin and may have different fiber orientation. The fiber orientation, stacking arrangements and material properties influence the performance of the laminate [12]. The following assumptions are made in formulations: (i) the middle plane

of the plate is taken as the reference plane. (ii) The laminated plate consists of arbitrary number of homogeneous, linearly elastic orthotropic layers perfectly bonded to each other. (iii) The lateral displacements are small compared to plate thickness. (iv) Normal strain in z -direction is neglected [12].

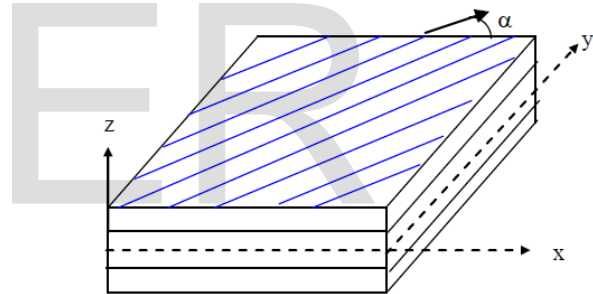


Fig 1:-Composite Plate

Manufacturing of composite material is an important step in manufacturing of composite driveshaft as it greatly affects the properties of composite material. Composite material consists of strong fibers either continuous or non-continuous embedded in weaker material matrix. The matrix transfers load on the composite component to the strong fibers and this matrix keeps the geometric arrangement of fibers. This resulting composition is capable of efficient mechanical performance. There are many factors that affect the composite material properties such as fibers, materials for matrices and resins.

2 SELECTION OF MATERIAL

A. Selection of Cross-Section for the driveshaft

For the selection of cross section of the drive shaft solid circular or hollow circular are generally considered. Here

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hollow circular cross section is chosen due to following advantages.

1. The solid circular shafts are weaker in per kg weight than the hollow circular shaft.
2. In case of the solid circular shaft, the stress distribution at the outer surface is more and at center it is zero. But in hollow circular shaft, stress variation is smaller
3. Also in solid circular shafts, material closer to the center is not fully utilized.

B. Selection of Reinforcement Fiber for the driveshaft

The different types of fibers have different properties. The selection of fiber for design of shaft depends on the physical properties and performance requirements. Fiber composites consist of matrices reinforced by short (discontinuous) or long (continuous) fibers. Fibers are generally anisotropic and examples include carbon and aramids. Glass is the most common fiber used in polymer matrix composites. Its advantages include its high strength, low cost, high chemical resistance, and good insulating properties. The drawbacks include low elastic modulus, poor adhesion to polymers, high specific gravity, sensitivity to abrasion (reduces tensile strength), and low fatigue strength. The main types are E-glass (also called "fiberglass") and S-glass. The "E" in E-glass stands for electrical because it was designed for electrical applications. However, it is used for many other purposes now, such as decorations and structural applications. The "S" in S-glass stands for higher content of silica. It retains its strength at high temperatures compared to E-glass and has higher fatigue strength. Graphite fibers are very common in high-modulus and high-strength applications such as aircraft components, etc. The advantages of graphite fibers include high specific strength and modulus, low coefficient of thermal expansion, and high fatigue strength. The drawbacks include high cost, low impact resistance, and high electrical conductivity. Epoxy resins are the most commonly used resins. They are low molecular weight organic liquids containing epoxide groups. Epoxide has three members in its ring: one oxygen and two carbon atoms.

				tensio n		
Carbon fiber HM	385	20	0.23	3630	0.4	2170
E-Glass fiber	72	27.7	0.3	3450	4.7	2580
S-Glass fiber	87	33.5	0.3	4710	5.6	2460
Kevlar 49 fiber	124	5	0.3	3850	2.8	1440
Steel	206	81	0.27	648	4	7800
Alumin um	69	25.6	0.35	234	3.5	2600

C. Selection of the resin system

The mixture or formulation of the polymer and polymer precursor material with various additives or chemically reactive components is known as the Resin. The processing, fabrication and the ultimate properties of composite material will affect the chemical composition and physical properties of the resin. The handling ability and processing ability of the composites may be affected by the variation in the composition, physical state or morphology of a resin and presence of impurities or contaminants in a resin. Also, it affects the long term durability of the composite material and the properties of lamina/laminate.

The main factors considered in the selecting resin are resistance to impact (a function of modulus of elongation), elongation to failure, temperature capability and the important one is cost of the resin. The commonly used resins for the composite driveshaft are either epoxies or the vinyl esters. The following table shows the mechanical properties of the resins used for the manufacturing of the composite material.

TABLE 1: MECHANICAL PROPERTIES OF FIBERS[11]

Materia l	Youn g's modu lus (GPa)	Shear modu lus (Gpa)	Axial Poisso n's ratio	Ultim ate streng th (Mpa)	Strai n to failu re (%)	Densi ty (Kg/ m3)

TABLE 2 : MECHANICAL PROPERTIES OF THE RESINS[11]

Materi al	Youn g's modu lus (GPa)	Shear modu lus (GPa)	Axial Poisso n's ratio	Ultim ate streng th (MPa) tensio	Strai n to failu re (%)	Densit y (Kg/ m3)

				n		
Epoxy	3.1	1.2	0.3	70	4.0	1200
Polyester	3.5	1.4	0.3	70	5.0	1100

Here, epoxy resin is selected due to its strength, good wetting of fibers and lower curing shrinkage.

3 DESIGN OF COMPOSITE DRIVESHAFT

The torque transmission capability of the driveshaft should be larger than the 3,500 Nm and the fundamental natural bending frequency of the passenger cars should be more than the 80 Hz i.e.it should be more than the minimum natural bending frequency. The outer diameter of the driveshaft should be such that it should not exceed 100 mm due to space limitation. Here, the outer diameter of the shaft is taken as 50 mm.

TABLE 3.DESIGN REQUIREMENTS AND SPECIFICATIONS

Sr.No.	Name	Notation	Unit	Value
1.	Ultimate Torque	Tmax	Nm	3500
2.	Max. Speed of Shaft	Nmax	rpm	5700
3.	Length of shaft	L	mm	660

While designing a composite driveshaft, the important aspects which are needed to be considered based on the literature and available standards of automotive driveshaft are as follows. Here, the composite driveshaft is designed for the specific application of passenger cars. The selected model of the passenger car is Maruti Suzuki Omni vehicle.

A. Torsional Strength

The maximum shear stress of the shaft will be calculated by,

$$\tau_{max} = \frac{T}{2\pi r_m^2 t}$$

where,

r_m is the mean radius of the shaft.

Thus, the mean radius of the shaft will be calculated by,

$$r_m = r_o - \frac{t}{2}$$

$$r_m = 25 - \frac{20}{2}$$

$$r_m = 15 \text{ mm}$$

Therefore, putting this value in shear stress equation, we get,

$$\tau_{max} = \frac{T}{2\pi r_m^2 t}$$

$$\tau_{max} = \frac{3500 \times 10^3}{2\pi \times 15^2 \times 20}$$

$$\tau_{max} = 123.78 \text{ N/mm}^2$$

Torsional buckling strength

Considering the hollow composite shaft as anisotropic cylindrical shell, the buckling torque is given by:

$$T_b = 2\pi r^2 t \times 0.272 \times (E_x \times E_y^3)^{\frac{1}{4}} \left(\frac{t}{r}\right)^{\frac{3}{2}} \dots \dots \dots (1)$$

where,

E_x = Young's modulus in 'x' direction

E_y = Young's modulus in 'y' direction

Here, we considered the composite driveshaft as orthotropic lamina.

So, for orthotropic lamina, the longitudinal elastic modulus will be calculated by following formula,

$$\frac{1}{E_x} = \frac{1}{E_1} \times (\cos \theta)^4 + \left(\frac{1}{G_{12}} - \frac{2\theta_{12}}{E_1}\right) \times (\sin \theta)^2 (\cos \theta)^2 + \frac{1}{E_2} (\sin \theta)^4 \dots \dots \dots (2)$$

$$\frac{1}{E_y} = \frac{1}{E_1} \times (\sin \theta)^4 + \left(\frac{1}{G_{12}} - \frac{2\theta_{12}}{E_1}\right) \times (\sin \theta)^2 (\cos \theta)^2 + \frac{1}{E_2} (\cos \theta)^4 \dots \dots \dots (3)$$

The values of E_x and E_y will be found out with the help of E_1, E_2, G_{12} & θ_{12} values.

The value of θ i.e. Stacking sequence angle will be taken so that the torsional buckling strength will be more than that of the maximum torque applied.

TABLE 4. PROPERTIES OF FIBER/EPOXY PLIES[11]

For Fiber volume fraction Vf=0.6	Glass	Kevlar	Carbon
Specific mass, ρ (kg/m ³)	2,080	1,350	1,530
Longitudinal tensile	1,250	1,410	1,270

strength, $\sigma_{l\ rupture}^{tensile}$ (MPa)			
Longitudinal compressive strength, $\sigma_{l\ rupture}^{comp}$ (MPa)	600	280	1,130
Transverse tensile strength, $\sigma_{t\ rupture}^{tensile}$ (MPa)	35	28	42
Transverse Compressive strength, $\sigma_{t\ rupture}^{comp}$ (MPa)	141	141	141
In-plane shear strength, $\tau_{lt\ rupture}$ (MPa)	63	45	63
Interlaminar shear strength, $\tau_{lt\ rupture} = \tau_{tl\ rupture}$ (MPa)	80	60	90
Longitudinal elastic modulus E_l (MPa)	45,000	85,000	134,000
Transverse elastic modulus E_t (MPa)	12,000	5,600	7,000
Shear modulus G_{lt} (MPa)	4,500	2,100	4,200
Poisson's Ratio ν_{lt} (MPa)	0.3	0.34	0.25
Longitudinal coefficient of thermal expansion at 200C, α_l (0C-1)	0.4- 0.7× 10 ⁻⁵	- 0.4× 10 ⁻⁵	- 0.12× 10 ⁻⁵

$$T_b = 2\pi r^2 t \times 0.272 \times (E_x \times E_y^3)^{\frac{1}{4}} \times \left(\frac{t}{r}\right)^{\frac{3}{2}}$$

$$T_b = 2\pi \times 15^2 \times 20 \times 0.272 \times (38709.5 \times 38709.5^3)^{\frac{1}{4}} \times \left(\frac{20}{15}\right)^{\frac{3}{2}}$$

$$T_b = 4583.39 \text{ Nm}$$

Here, $T_b > T$

Therefore, the design is safe.

C. Natural frequency

The bending natural frequency of the shaft is given by.

$$f_{nb} = \frac{\pi}{2} \sqrt{\frac{E_x I_x}{m' L^4}}$$

Here,

The moment of inertia of hollow shaft is given by,

$$I_x = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$I_x = \frac{\pi}{64} (50^4 - 10^4)$$

$$I_x = 3.0630 \times 10^{-7} \text{ m}^4$$

The mass per unit length of the shaft is given by,

$$m' = \rho \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$m' = \rho \frac{\pi}{4} (0.050^2 - 0.010^2)$$

$$m' = 3.92 \text{ kg/m}$$

Hence,

$$I_x = 0.18 \times 10^{-6} \text{ m}^4$$

and $\rho = 2.88 \text{ kg/m}$.

Upon substitution, the fundamental bending natural frequency is,

$$f_{nb} = \frac{\pi}{2} \sqrt{\frac{E_x I_x}{m' L^4}}$$

$$f_{nb} = \frac{\pi}{2} \sqrt{\frac{38709.5 \times 3.0630 \times 10^{-7}}{3.92 \times 0.66^4}}$$

$$f_{nb} = 198.32 \text{ Hz} (> 80 \text{ Hz})$$

Here, the fundamental bending natural frequency of composite shaft is greater than the minimum natural frequency of the composite shaft assumed.

Therefore, the designed composite shaft is safe.

D. Critical Speed of the Composite Shaft

The critical speed of the shaft is given by,

$$\text{Critical speed} = \frac{60 \times \pi^2}{2\pi \times L^2} \sqrt{\frac{EI}{\rho A}}$$

Where,

E = Modulus of elasticity,

D = Diameter of shaft,

ρ = Density,

W = Total weight of the shaft,

L = Length of the shaft,

$$I_x = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$I_x = 3.0630 \times 10^{-7} \text{m}^4$$

$$A = \frac{\pi}{4}(d_o^2 - d_f^2)$$

$$A = 1.885 \times 10^{-3} \text{m}^2$$

Now, Put in above equation,

$$\text{Critical speed} = \frac{60 \times \pi^2}{2\pi \times 0.660^2} \sqrt{\frac{385 \times 10^6 \times 3.0630 \times 10^{-7}}{2080 \times 10^{-9} \times 1.885 \times 10^{-3}}}$$

$$\text{Critical speed} = 17837.68 \text{ rpm}$$

E. Weight Calculation of Composite Shaft

Weight of Composite drive shaft:-

$$\text{Weight} = \text{Density} \times \text{Volume}$$

The density of glass/epoxy fiber is 2080 kg/m³.

$$W = \rho \times A \times L$$

$$W = 2080 \times 10^{-9} \times \frac{\pi}{4}(d_o^2 - d_f^2) \times L$$

$$W = 2080 \times 10^{-9} \times \frac{\pi}{4}(50^2 - 10^2) \times 660$$

$$W = 1.45 \text{ Kg}$$

The weight of the composite driveshaft is 1.45 kg.

4 OPIMIZATION OF WINDING ANGLE

Most multi-objective optimization studies have been focused on nature-inspired algorithms. Many nature-inspired optimization algorithms have been proposed, such as the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Ant Colony Optimization (ACO), Harmony Search (HS), the Grenade-Explosion Method (GEM), etc.: these approaches are based on different natural phenomena. GA uses the theory of Darwin based on the survival of the fittest, PSO implements the foraging behavior of a bird searching for food, and ABC uses the foraging behavior of a honey bee. The real world features many problems for which optimizing two or more objective functions simultaneously is desirable. All of the evolutionary- and swarm intelligence-based algorithms are probabilistic algorithms and require common controlling parameters, like the population size, number of generations, elite size, etc. In addition to the common control parameters, algorithm-specific control-parameters are required. For example, GA uses the mutation rate and crossover rate. Similarly, PSO uses the inertia weight, as well as social and cognitive parameters. The improper tuning of algorithm-specific parameters either increases the computational effort or yields a local optimal solution, Therefore, Rao et al. recently introduced the teaching-learning based optimization

(TLBO) algorithm, which requires only the common control parameters and does not require any algorithm-specific control parameters. Other evolutionary algorithms require the control of common control parameters as well as the control of algorithm-specific parameters. The burden of tuning control parameters is comparatively less in the TLBO algorithm. Thus, the TLBO algorithm is simple, effective and involves comparatively less computational effort [6].

Teaching Learning Based Optimization was proposed by R.V. Rao in 2011. It is considered as a population based method as it uses a population of solutions in order to obtain the global solution. The process of TLBO consists of two parts: the first part is the “teacher phase” and the second part is the “Learners phase”. The “teacher phase” means learner learns from the teacher and the “Learners phase” means learner learns by the interaction between themselves. In this optimization algorithm a group of learners is considered as population and different subjects offered to the learners are considered as different design variables of the optimization problem and a learner’s result is analogous to the ‘fitness’ value of the optimization problem. The best solution in the entire population is considered as the teacher. The design variables are actually the parameters involved in the objective function of the given optimization problem and the best solution is the best value of the objective function. The working of TLBO is divided into two parts, ‘Teacher phase’ and ‘Learner phase’.

Teacher phase:

During this phase a teacher tries to increase the mean result of the class in the subject taught by him or her depending on his or her capability. At any iteration *i*, assume that there are ‘*m*’ number of subjects (i.e. design variables), ‘*n*’ number of learners (i.e. population size, *k*=1,2,...,*n*) and *M_{j,i}* be the mean result of the learners in a particular subject ‘*j*’ (*j*=1,2,...,*m*). The best overall result *X_{total}* - *kbest_i*, considering all the subjects together obtained in the entire population of learners can be considered as the result of best learner *kbest_i*. However, as the teacher is usually considered as a highly learned person who trains learners so that they can have better results, the best learner identified is considered by the algorithm as the teacher. The difference between the existing mean result of each subject and the corresponding result of the teacher for each subject is given by,

$$\text{Difference_Mean}_{j,k,i} = r_i(X_{j,kbest,i} - TFM_{j,i}),$$

Where, *X_{j,kbest,i}* is the result of the best learner (i.e. teacher) in subject *j*. *TF* is the teaching factor which decides the value of mean to be changed, and *r_i* is the random number in the range [0, 1]. Value of *TF* can be either 1 or 2. The value of *TF* is decided randomly with equal probability as,

$$TF = \text{round} [1 + \text{rand}(0,1)\{2-1\}]$$

TF is not a parameter of the TLBO algorithm. The value of TF is not given as an input to the algorithm and its value is randomly decided by the algorithm using Eq. (2). After conducting a number of experiments on many benchmark functions it is concluded that the algorithm performs better if the value of TF is between 1 and 2. However, the algorithm is found to perform much better if the value of TF is either 1 or 2 and hence to simplify the algorithm, the teaching factor is suggested to take either 1 or 2 depending on the rounding up criteria given by Eq.(2). Based on the Difference_Mean_{j,k,i}, the existing solution is updated in the teacher phase according to the following expression.

$$X'_{j,k,i} = X_{j,k,i} + \text{Difference_Mean}_{j,k,i}$$

Where $X'_{j,k,i}$ is the updated value of $X_{j,k,i}$. Accept $X'_{j,k,i}$ if it gives better function value. All the accepted function values at the end of the teacher phase are maintained and these values become the input to the learner phase. The learner phase depends upon the teacher phase.

Learner phase:

Learners increase their knowledge by interaction among themselves. A learner interacts randomly with other learners for enhancing his or her knowledge. A learner learns new things if the other learner has more knowledge than him or her. Considering a population size of 'n', the learning phenomenon of this phase is expressed below. Randomly select two learners P and Q such that $X'_{\text{total-P},i} \neq X'_{\text{total-Q},i}$ (where, $X'_{\text{total-P},i}$ and $X'_{\text{total-Q},i}$ are the updated values of $X_{\text{total-P},i}$ and $X_{\text{total-Q},i}$ respectively at the end of teacher phase)

$X'_{j,P,i} = X'_{j,P,i} + r_i(X'_{j,P,i} - X'_{j,Q,i})$, If $X'_{\text{total-P},i} < X'_{\text{total-Q},i}$
 $X'_{j,P,i} = X'_{j,P,i} + r_i(X'_{j,Q,i} - X'_{j,P,i})$, If $X'_{\text{total-Q},i} < X'_{\text{total-P},i}$
 Accept $X'_{j,P,i}$ if it gives a better function value.

Steps for implementing TLBO

TLBO can be implemented easily, just by following the below steps:

Step 1: Define the optimization problem and initialize the optimization parameter. Initialize the population size, number of generations and number of design variable and limit of design variables.

Step 2: Generate a random population as per the population size and the design variables. Population size denotes the no. of learners and the design variable denotes the subject. The population is expressed as,

Step 3: Teacher phase:

- a) Calculate the mean of the population column wise, which will give the mean for the particular subject as,

$$M_D = [m_1 m_2 \dots \dots \dots m_D]$$

- b) The best solution will act as teacher for that particular iteration.

$$X_{\text{teacher}} = X_{f(x)=\min}$$

- c) The teacher will try to move the mean from M_D towards X_{teacher} which will act as a new mean from that iteration.

$$M_{\text{new},D} = X_{\text{teacher},D}$$

- d) The difference between two mean is expressed as

$$\text{Difference}_D = (M_{\text{new},D} - T_f M_D)$$

Where, $T_f = \text{round} [1 + \text{rand} (0, 1)]\{1,2\}$

- e) The value of T_f is selected as 1 or 2. The obtained difference is added to the current solution to update its value using

$$X_{\text{new},D} = X_{\text{new},D} + \text{difference}_D$$

- f) X_{new} Is accepted if it gives better function value.

Step 4: Learner phase

In this phase the learner increase the knowledge with interaction.

Step 5: Termination Criteria

If the maximum generation number is achieved, stop the algorithm or repeat from step 3 i.e. from teacher phase.

Here,

Population size = 180

Range = $-90 \leq \theta \leq 90$

Design Variables = 1

Objective Function =

$$\frac{1}{E_x} = \frac{1}{E_1} \times (\cos \theta)^4 + \left(\frac{1}{G_{12}} - \frac{2\theta_{12}}{E_1} \right) \times (\sin \theta)^2 (\cos \theta)^2 + \frac{1}{E_2} (\sin \theta)^4$$

Table 5. Initial Population

θ	EX	EY	Tb
30	17590	11440	26664
55	11625	15356	29981
75	11672	30515	50230
85	11956	42608	64908
90	12000	45000	67686
Mean=67			

$$\text{Diff.Mean} = 0.58 * (90-67) = 13.34$$

Now, in learner phase random interaction is considered amongst the learners. Then after interaction values of EX and EY get updated depending upon learners capacity.

As Tb value for learner 56.73 is better, this value is selected.

Hence the optimum angle obtained is 56.73.

5 CONCLUSION

It is concluded that with the use of composite driveshaft weight of the driveshaft gets reduced which gives less fuel consumption. Optimization of certain parameters like winding angle, diameter can give increased performance. Therefore conventional two-piece steel driveshaft can be replaced by composite driveshaft.

REFERENCES

- [1] Sumana B. G. et. al. "Investigation of Burst Pressure on Carbon Glass Fiber Reinforced Polymer Metal Tube for High Pressure Applications", *Procedia Materials Science*, 2014.
- [2] Aleksandr Cherniaev & Valeriy Komarov "Multistep Optimization of Composite Drive Shaft Subject to Strength, Buckling, Vibration and Manufacturing Constraints", *Application of composite material*, Springer Science, 2014.
- [3] S.A. Mutasher "Prediction of the torsional strength of the hybrid aluminum/composite drive shaft", *Materials and Design*, Elsevier, 2008.
- [4] A.R. Abu Talib et. al. "Developing a hybrid, carbon glass fiber-reinforced epoxy composite automotive drive shaft", *Materials and Design*, Elsevier, 2010.
- [5] M.A. Badie et. al. "An investigation into hybrid carbon/glass fiber reinforced epoxy composite automotive drive shaft", *Materials and Design*, Elsevier, 2011.
- [6] R. V. Rao "TLBO algorithm and its engineering applications", Springer Publications, 2016.
- [7] D. G. Lee, "Design and manufacture of an automotive hybrid aluminum/composite drive shaft", *Composite structures*, Elsevier Publications, 2004.
- [8] O. Motangier and C. Hochard, "Optimisation of hybrid high-modulus/high-strength carbon fibre reinforced plastic composite drive shafts", *Materials and design*, Elsevier Publications 2013.
- [9] R. Sino et. al., "Dynamic analysis of a rotating composite shaft", *Composite Science and technology*, Elsevier 2007.
- [10] Motangier and C. Hochard, "Dynamics of a supercritical composite shaft mounted on viscoelastic supports", *Journal of sound and vibration*, Elsevier 2013.
- [11] Autar K. Kaw, "Mechanics of Composite Materials", Taylor and Francis group, CRC press, 2013, pp.411-420.
- [12] D. Gay, "Composite Materials Design and Applications", CRC press Taylor and Francis Group 2015.